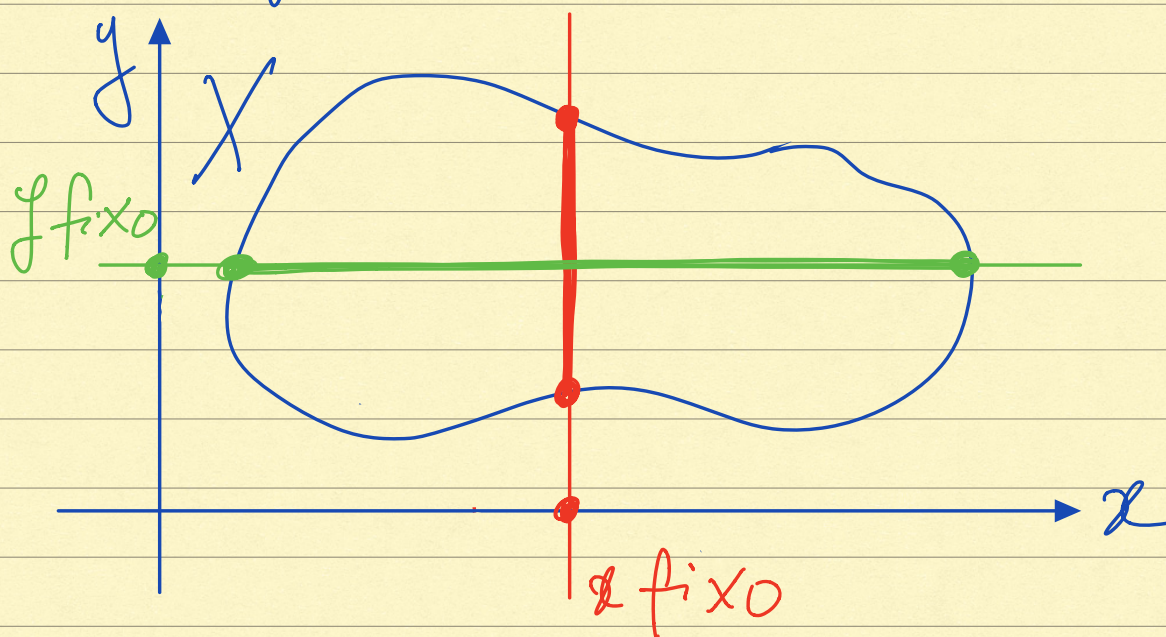


Integrais Múltiplos. Área em \mathbb{R}^2

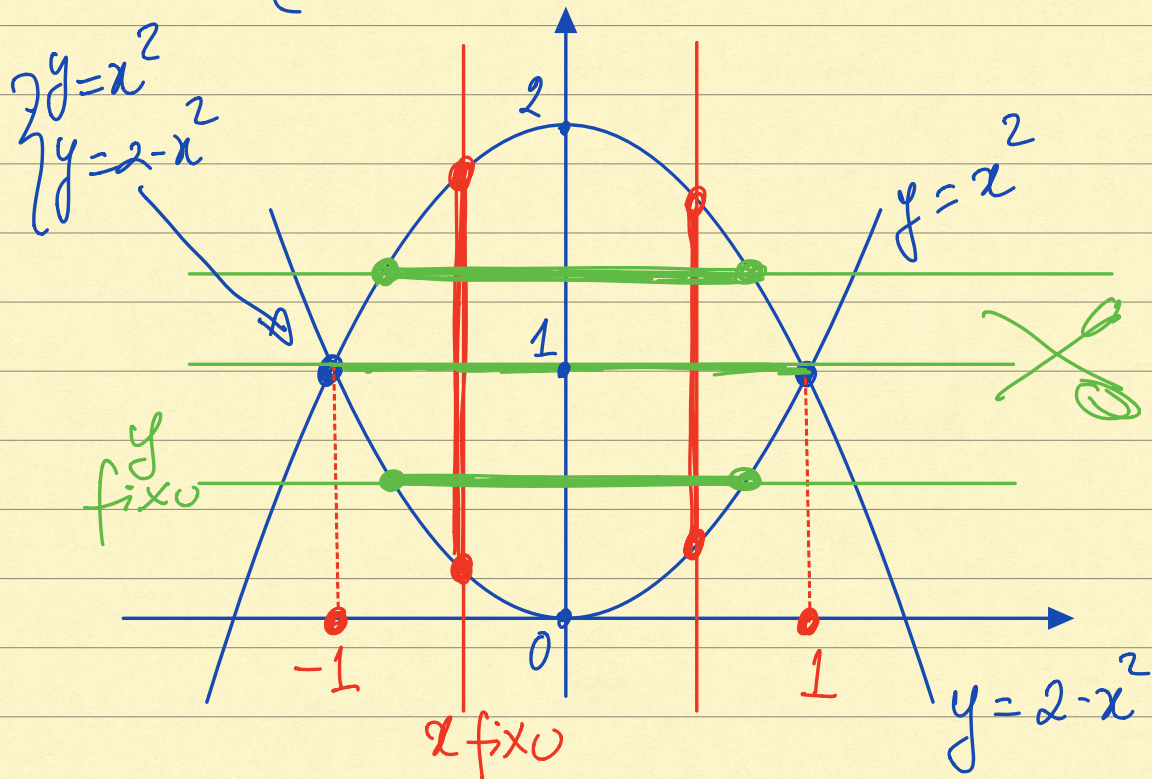


$$y \text{ fixo} \rightarrow x(y) \quad \boxed{dx dy}$$
$$\text{Área de } X = \int_c^d \left(\int_{a(y)}^{b(y)} 1 dx \right) dy$$

$$x \text{ fixo} \rightarrow y(x) \quad \boxed{dy dx}$$
$$\text{Área de } X = \int_a^b \left(\int_{c(x)}^{d(x)} 1 dy \right) dx$$

Exemplo:

$$X = \left\{ (x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 2 - x^2 \right\}$$



x fixo $\longrightarrow y(x)$

$$\text{Área de } X: \int_{-1}^1 \left(\int_{x^2}^{2-x^2} 1 \, dy \right) dx =$$

$$= \int_{-1}^1 (2 - x^2 - x^2) dx = \int_{-1}^1 (2 - 2x^2) dx$$

$$= 2 \int_0^1 (2 - 2x^2) dx = 2 \left(2 - \frac{2}{3} \right)$$

$$= \frac{8}{3}$$

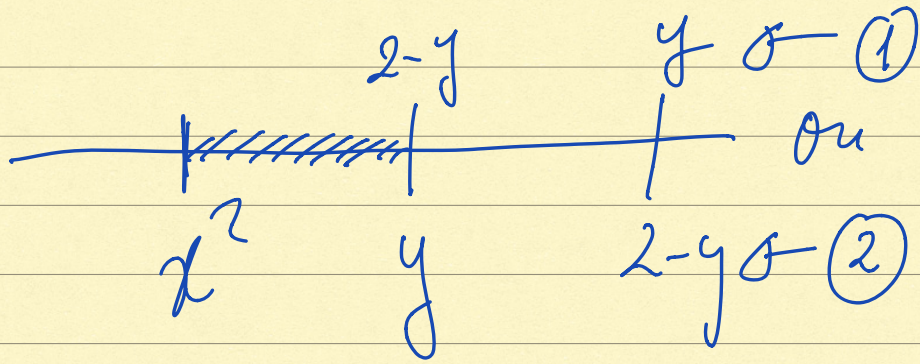
y fixo \rightarrow $x(y)$ (see figure)

$$X: 0 \leq x^2 \leq y \leq 2 - x^2 \leq 2$$

$$0 \leq y \leq 2$$

$$x^2 + y \leq 2$$

$$\left\{ \begin{array}{l} x^2 \leq y \\ x^2 \leq 2 - y \end{array} \right.$$



$$\textcircled{1} \quad x^2 < \boxed{2-y < y}$$

$$2 < y+y$$

$$2 < 2y$$

$$\boxed{1 < y}$$

$$\boxed{1 < y < 2}$$

$$\rightarrow \boxed{x^2 < 2-y}$$

$$-\sqrt{2-y} < x < \sqrt{2-y}$$

$$\textcircled{2} \quad x^2 < y < 2-y$$

$$\boxed{y < 1}$$

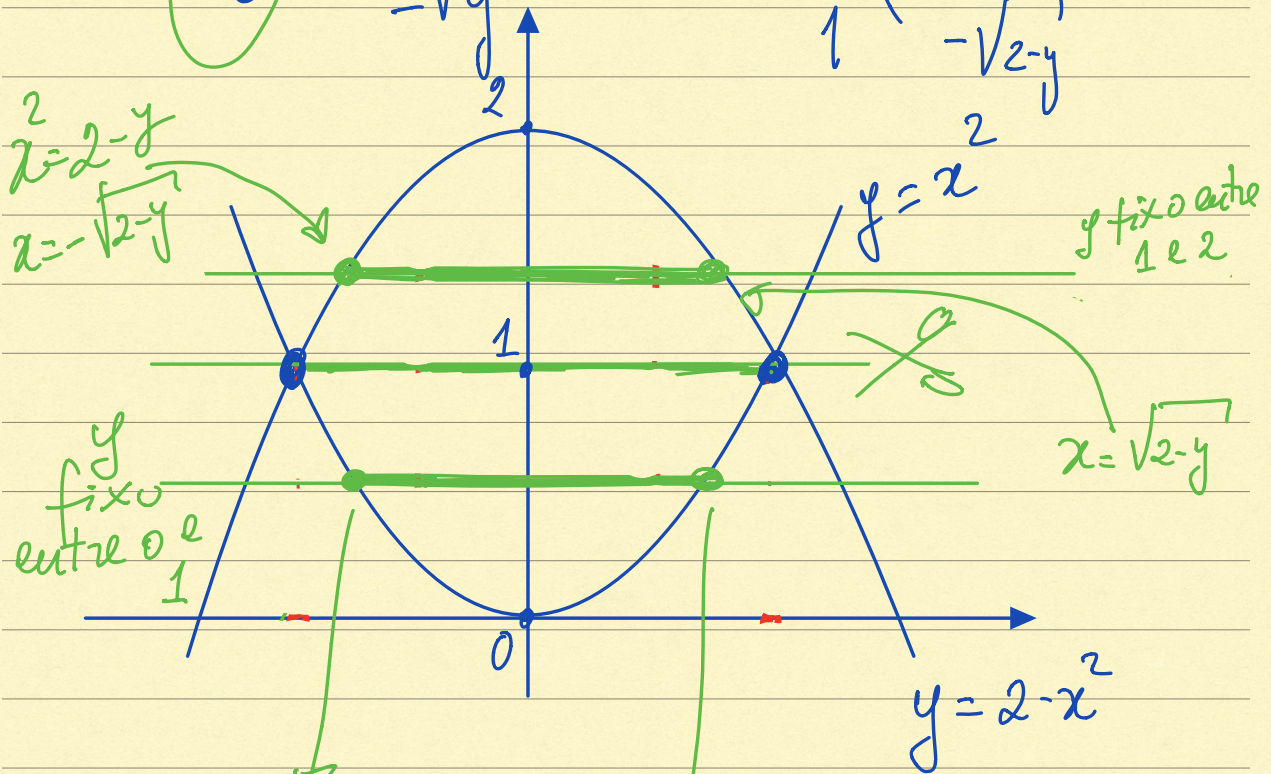
$$\rightarrow \boxed{x^2 < y}$$

$$\boxed{0 < y < 1}$$

$$\rightarrow -\sqrt{y} < x < \sqrt{y}$$

Area de X:

$$\int_0^1 \left(\int_{-\sqrt{y}}^{\sqrt{y}} dx \right) dy + \int_1^2 \left(\int_{-\sqrt{2-y}}^{\sqrt{2-y}} dx \right) dy$$



$$x^2 = y \Rightarrow x = -\sqrt{y} \vee x = \sqrt{y}$$

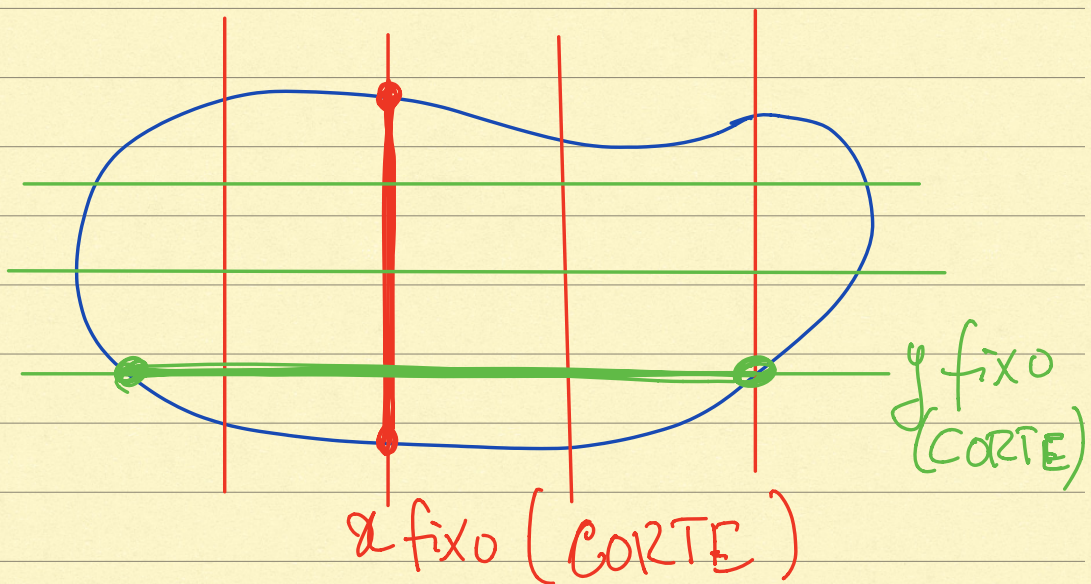
Mais fácil c/ figure !!!

1- Descrever o conjunto através de inequações:

Exemplo: $x^2 < y < 2 - x^2$.

2- Esboçar a figura.

3- Fixar x ou y : (CORTES)



4- Partir, se necessário, a figura em várias "partes".